

A STUDY OF QUADRATURE
BENDING MODE SIGNAL CANCELLATION
BY THE USE OF RC COMMUTATED NETWORKS

Prepared by

SERVO MECHANISMS LABORATORY

C. L. PHILLIPS, PROJECT LEADER

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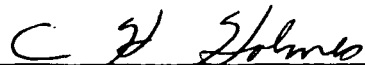
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APPROVED BY:


C. H. Holmes, Head Professor
Electrical Engineering
Department

SUBMITTED BY:



C. L. Phillips, Professor
Electrical Engineering
Department

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FOREWORD

This document is a final technical summary of the progress made by the Electrical Engineering Department of Auburn University in the final phase of Contract NAS8-11116. This contract was granted to Auburn Research Foundation, Auburn, Alabama, October 21, 1963, by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama, and was modified November 21, 1964.

SUMMARY

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The development of an approximate steady-state time response expression for an RC commutated network is given. The expression is used in investigating several modifications of a rate gyro blender which utilize RC commutated networks. The RC commutated networks were added to the rate gyro blender system to attain more complete signal cancellation irrespective of the phase relation between the forward and aft input signals. Total cancellation is achieved by cancellation of the inphase and quadrature components separately. The responses of the modifications to various sinusoidal input signals containing quadrature components are compared experimentally.

Author

PERSONNEL

The following named persons have actively participated on contract
NAS8-11116:

- C. H. Holmes - Head Professor of Electrical Engineering
- C. H. Weaver - Former Head Professor of Electrical Engineering
- G. T. Nichols - Associate Professor of Electrical Engineering
- H. M. Summer - Former Professor of Electrical Engineering
- C. L. Phillips - Professor of Electrical Engineering
- W. M. Feaster - Associate Professor of Electrical Engineering
- J. L. Lowry - Professor of Electrical Engineering
- W. L. McDaniel, Jr. - Former Instructor in Electrical Engineering
- D. L. Chenoweth - Graduate Assistant in Electrical Engineering
- T. S. Craven - Former Graduate Assistant in Electrical Engineering
- F. C. Roan - Former Graduate Assistant in Electrical Engineering

I. INTRODUCTION

This report is the final technical report on Phase II of Contract NAS8-11116. The objective of the work done under this phase of the contract was the investigation of the use of RC commutated networks for the purpose of eliminating the quadrature component of the first bending mode signal in a rate gyro blender.

The primary objective of this report is to present the mathematical studies which have been developed at Auburn University and delivered to NASA - Marshall Space Flight Center, the experimental results in connection with these studies, and the conclusions and recommendations resulting from the effort. Most of the material presented herein summarizes information contained in the previous monthly progress reports.

Chapter II contains a brief description of the operation of a rate gyro blender, and in Chapter III, a mathematical analysis of a certain type of RC commutated network is presented. In Chapter IV, various modifications of the rate gyro blender are investigated. Several of the modifications utilize RC commutated networks to improve input signal cancellation. Chapter V contains a mathematical analysis of a cascaded configuration of two RC commutated networks. Several recommendations are given in Chapter VI.

II. RATE GYRO BLENDER OPERATION¹

A rate gyro blender system was designed by Minniapolis-Honeywell Regulator Company and delivered to NASA - George C. Marshall Space Flight Center, Huntsville, Alabama, under Contract NAS8-5069. The system was designed for the purpose of eliminating the first bending mode signal from the Saturn V control system.

Throughout this report, the phrase "bending mode" refers only to the first bending mode. It is assumed that the higher frequency components are attenuated by proper filtering.

The configuration of the rate gyro blender is shown in Figure 1. The forward and aft input signals to the rate gyro blender are the outputs of two rate gyros, one located forward and the other aft of the first bending mode antinode of the space vehicle structure. The rate gyro signals contain undersirable bending mode signals as well as the desirable rigid body rate signals. The undersirable bending mode signals from the two rate gyros are nearly opposite in phase and have different amplitudes, while the rigid body rate signals are in phase and equal in amplitude.

If the bending mode signals are assumed to be 180° out of phase, the attenuators, K and $1-K$, are adaptively adjusted such that the attenuated bending mode signals are equal in amplitude, and therefore sum to zero. The rigid body rate signals are assumed to be equal in

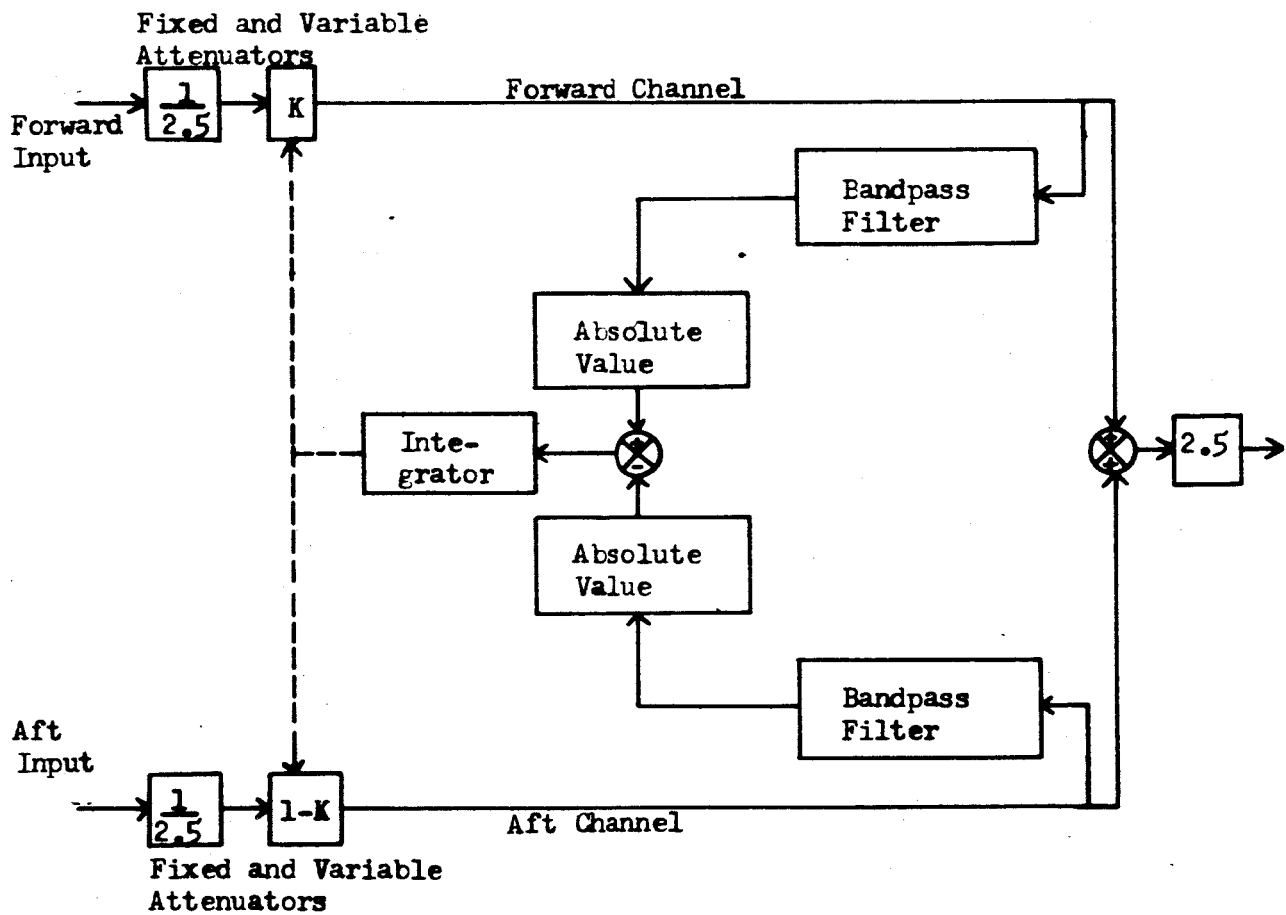


Fig. 1.-Rate Gyro Blender Configuration

amplitude and phase, and the sum of the attenuated signals is therefore equal to the original rate signal.

The adaptive attenuation adjustment is obtained by comparing the magnitudes of the first bending mode signals after the K and 1-K attenuators. In order to separate the first bending mode signals from the other system frequencies, the attenuator outputs are passed through bandpass filters with center frequencies fixed near the center of the first bending mode frequency range. The comparison of the attenuated bending mode signals is accomplished by means of absolute value circuits which follow the bandpass filters. An error signal, which results from the subtraction of the absolute values of attenuated first bending mode signals, is integrated and used to adjust the magnitude of K.

Because the forward and aft bending mode signals in reality are not always 180° out of phase, the attenuated first bending mode signals will not sum to zero, even though their amplitudes are made equal by suitable adjustment of K.

In order to make the blender fully effective, additional means must be improvised to insure complete bending mode cancellation regardless of the phase relationship between the forward and aft input signals. One method of accomplishing this is to operate on the inphase and quadrature components separately. This can be realized by additional filtering in the basic rate gyro blender. The additional filtering consists of single RC commutated networks as shown in Figure 2. The networks act as bandpass filters that pass only the component of the input signal in phase with the commutation function.

III. RC COMMUTATED NETWORK²

A time domain expression for the output of an RC commutated network with a sinusoidal input is derived, first for the case wherein the frequency of the input signal and that of the commutation signal are unequal, and then for the case wherein the frequencies are the same. An approximate Fourier series expansion of the output for the latter case is given, and then the sensitivity of the amplitude of the output fundamental to changes in the characteristics of the commutated network is discussed.

A. Introduction

Since the rate gyro blender is effective only if the forward and aft first bending mode signals are exactly 180° out of phase, it is necessary to eliminate any existing quadrature components of the signal by other means.

The RC commutated network illustrated in Figure 2 is one means for sensing the quadrature component in the first bending mode signal.

The network has the following properties:

In the case where the input is of the form $R \sin (\omega t + \phi)$ and the capacitor is commutated every π/ω seconds, the output fundamental component is nearly in phase with the commutation signal; and furthermore, its amplitude is proportional to the amplitude of the input component in phase with the commutation signal.

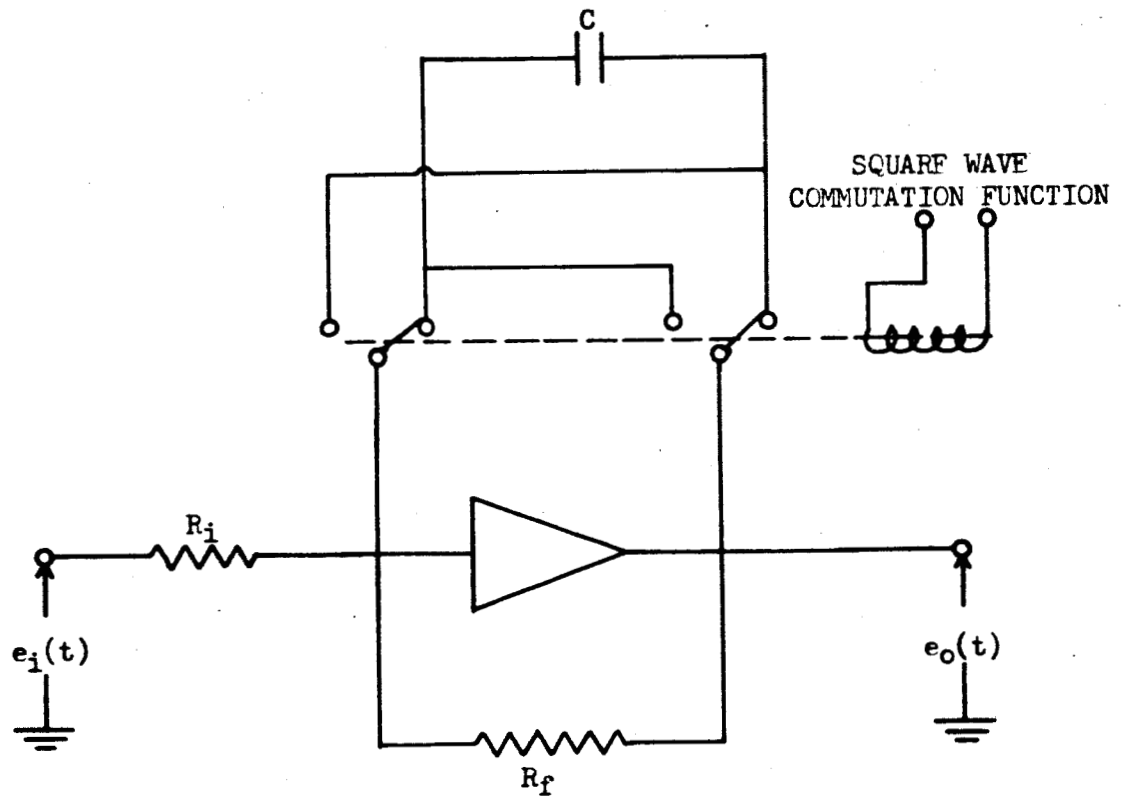


Fig. 2.-RC Commutated Network

In order to investigate possibilities for utilization of the RC commutated network, it is desirable to obtain a time expression for its response to a sinusoidal input during any commutation period. While a general transfer function of the network might in some cases be desirable, such a transfer function, by the strictest definition, does not exist for this network.

B. Response of the RC Commutated Network To a Sinusoidal Input

The RC commutated network, as illustrated in Figure 2, consists of a high gain amplifier with fixed input and feedback resistors and a commutated feedback capacitor. Each time the capacitor is reversed, the output voltage is also reversed, and thus a new initial charge on the capacitor must be considered following each commutation.

If an input $e_i(t)$ of the form

$$e_i(t) = R \sin(\omega t + \phi) \quad (\text{III-1})$$

is applied to the network, where R is the amplitude of the input, the differential equation describing the output of the RC commutated network between commutations is

$$\frac{de_o(t)}{dt} + ae_o(t) = b \sin(\omega t + \phi), \quad (\text{III-2})$$

where

$$a = \frac{1}{R_f C},$$

$$b = -\frac{R}{R_i C},$$

and $e_o(t)$ is the output voltage of the RC network. The solution of (III-2) is

$$e_o(t) = A e^{-at} + C_1 \sin(\omega t + \phi) + C_2 \cos(\omega t + \phi), \quad (\text{III-3})$$

where

$$C_1 = \frac{ab}{a^2 + \omega^2},$$

and

$$C_2 = \frac{-\omega b}{a^2 + \omega^2}.$$

Equation (III-3) can be written more conveniently as

$$e_o(t) = Ae^{-at} + \alpha \sin \omega t + \beta \cos \omega t, \quad (\text{III-4})$$

where

$$\alpha = C_1 \cos \phi - C_2 \sin \phi,$$

and

$$\beta = C_1 \sin \phi + C_2 \cos \phi.$$

The capacitor is commutated every $\frac{T}{2}$ seconds, (where T is the period of the square wave commutation function), and there is a new value of A, (the coefficient of the transient term in (III-4)), to be determined after each commutation.

The output expression for the RC commutated network can be derived for several switching periods in order to determine its general form during any future commutation period.

The first commutation is assumed to occur at $t = 0$. If the voltage across the capacitor just prior to the first commutation is

M volts, then the output voltage immediately after the first commutation is $-M$; and, $-M$ is the initial condition to be considered in computing the value of A from (III-4) during the time $0 < t < \frac{T}{2}$. Evaluating $e_o(0)^+$ from (III-4), and equating it to $-M$, yields

$$A = -M - \beta. \quad (\text{III-5})$$

The value of A can be substituted into (III-4) to obtain the output expression for $0 < t < \frac{T}{2}$, or

$$e_o(t) = \left[-(M + \beta)e^{-at} + \alpha \sin \omega t + \beta \cos \omega t \right]. \quad (\text{III-6})$$

$$\left[u(t) - u\left(t - \frac{T}{2}\right) \right].$$

The unit step function notation in (III-6) is used to denote the commutation period over which the output expression is valid.

The output voltage just prior to the second commutation can be evaluated from (III-6), and the value of A during the second commutation period can be obtained by equating the negative of this voltage to $e_o(\frac{T}{2})$, obtained from (III-4). If the new value of A is substituted into (III-4), an output expression for $\frac{T}{2} < t < T$ results, or

$$e_o(t) = \left\{ - \left[-(M + \beta) + 2(\alpha \sin \omega \frac{T}{2} + \beta \cos \omega \frac{T}{2}) e^{\frac{aT}{2}} \right] e^{-at} \right. \\ \left. + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \left[u(t - \frac{T}{2}) - u(t - T) \right]. \quad (\text{III-7})$$

The value of A can be calculated for $T < t < \frac{3T}{2}$ in the same manner as for the previous commutation periods. Then,

$$e_o(t) = \left\{ - \left[-(M + \beta) + 2(\alpha \sin \omega \frac{T}{2} + \beta \cos \omega \frac{T}{2}) e^{\frac{aT}{2}} \right. \right. \\ \left. \left. - 2(\alpha \sin \omega T + \beta \cos \omega T) e^{aT} \right] e^{-at} \right. \\ \left. + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \\ \left[u(t - T) - u(t - \frac{3T}{2}) \right]. \quad (\text{III-8})$$

By a similar procedure, the output for $\frac{3T}{2} < t < 2T$ is

$$e_o(t) = \left\{ - \left[-(M + \beta) + 2(\alpha \sin \omega \frac{T}{2} + \beta \cos \omega \frac{T}{2}) e^{\frac{aT}{2}} \right. \right. \\ \left. \left. - 2(\alpha \sin \omega T + \beta \cos \omega T) e^{aT} \right. \right. \\ \left. \left. + 2(\alpha \sin \omega \frac{3T}{2} + \beta \cos \omega \frac{3T}{2}) e^{\frac{3aT}{2}} \right] e^{-at} \right. \\ \left. + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \\ \left[u(t - \frac{3T}{2}) - u(t - 2T) \right]. \quad (\text{III-9})$$

By the process of mathematical induction, it can be seen that the output expression for any period $n \frac{T}{2} < t < (n+1) \frac{T}{2}$, where n is the number of commutations minus one, can be represented by

$$e_o(t) = \left\{ (-1)^n \left[-\frac{(M+\beta)-2}{2} \sum_{k=1}^n (-1)^k \left(\alpha \sin \omega k \frac{T}{2} + \beta \cos \omega k \frac{T}{2} \right) e^{ak \frac{T}{2}} \right] e^{-at} \right. \\ \left. + \alpha \sin \omega t + \beta \cos \omega t \right\}. \quad (\text{III-10})$$

$$\left[u\left(t - n \frac{T}{2}\right) - u\left(t - (n+1) \frac{T}{2}\right) \right].$$

C. Approximate Steady-State Output Expression For Input
And Commutation Function Equal in Frequency

The case wherein the input and commutation frequencies are equal can be analyzed as a special case of (III-10).

If, in (III-10), we let the input frequency ω be equal to the frequency of the commutation function, or $\omega = \frac{2\pi}{T}$, then (III-10) can be rewritten

$$e_o(t) = \left\{ (-1)^n \left[(-M + \beta) - 2\beta \sum_{k=0}^n e^{ak\frac{T}{2}} \right] e^{-at} + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \quad (III-11)$$

$$\left[u\left(t - \frac{nT}{2}\right) - u\left(t - (n+1)\frac{T}{2}\right) \right],$$

where n equals the number of commutations minus one. By the use of the relation,

$$\sum_{k=0}^n e^{ka\frac{T}{2}} = \frac{1 - e^{a(n+1)\frac{T}{2}}}{1 - e^{a\frac{T}{2}}}, \quad (III-12)$$

(III-11) can be rewritten as

$$e_o(t) = \left\{ (-1)^n \left[(-M + \beta) - 2\beta \frac{1 - e^{a(n+1)\frac{T}{2}}}{1 - e^{a\frac{T}{2}}} \right] e^{-at} + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \quad (III-13)$$

$$\left[u\left(t - \frac{T}{2}\right) - u\left(t - (n+1)\frac{T}{2}\right) \right].$$

If t becomes large enough so that $\left[-M + \beta - \frac{2\beta}{1 - e^{-\frac{aT}{2}}} \right] e^{-at}$

can be neglected in the expansion of (III-13), then the resulting steady-state approximation of $e_o(t)$ is

$$e_o(t) = \left\{ (-1)^n \left[\frac{2\beta e^{-a(t - n\frac{T}{2})}}{e^{-\frac{aT}{2}} - 1} \right] + \alpha \sin \omega t + \beta \cos \omega t \right\} \cdot \left[u(t - n\frac{T}{2}) - u(t - (n+1)\frac{T}{2}) \right] \quad (\text{III-14})$$

Because the difference between the actual response and the approximate steady-state response is $\left[-M + \beta - \frac{2\beta}{1 - e^{-\frac{aT}{2}}} \right] e^{-at}$,

the magnitude of $a = \frac{1}{R_f C}$ can be used as a measure of the response time of the RC commutated network.

As can be seen in (III-14), the steady-state response is independent of the voltage M on the capacitor prior to the first commutation.

In Figure 3, an experimentally determined steady-state response is compared with the derived response from (III-14) with a given set of input conditions. The purpose of this comparison is to verify the approximate steady-state response expression.

D. Fourier Series Expansion of Steady-State Output Expression

Since the approximate steady-state response given by (III-14) is finite, periodic, single valued, and has a finite number of finite

$$\frac{R_f}{R_i} = 1.23$$

$$C = 0.5 \mu f$$

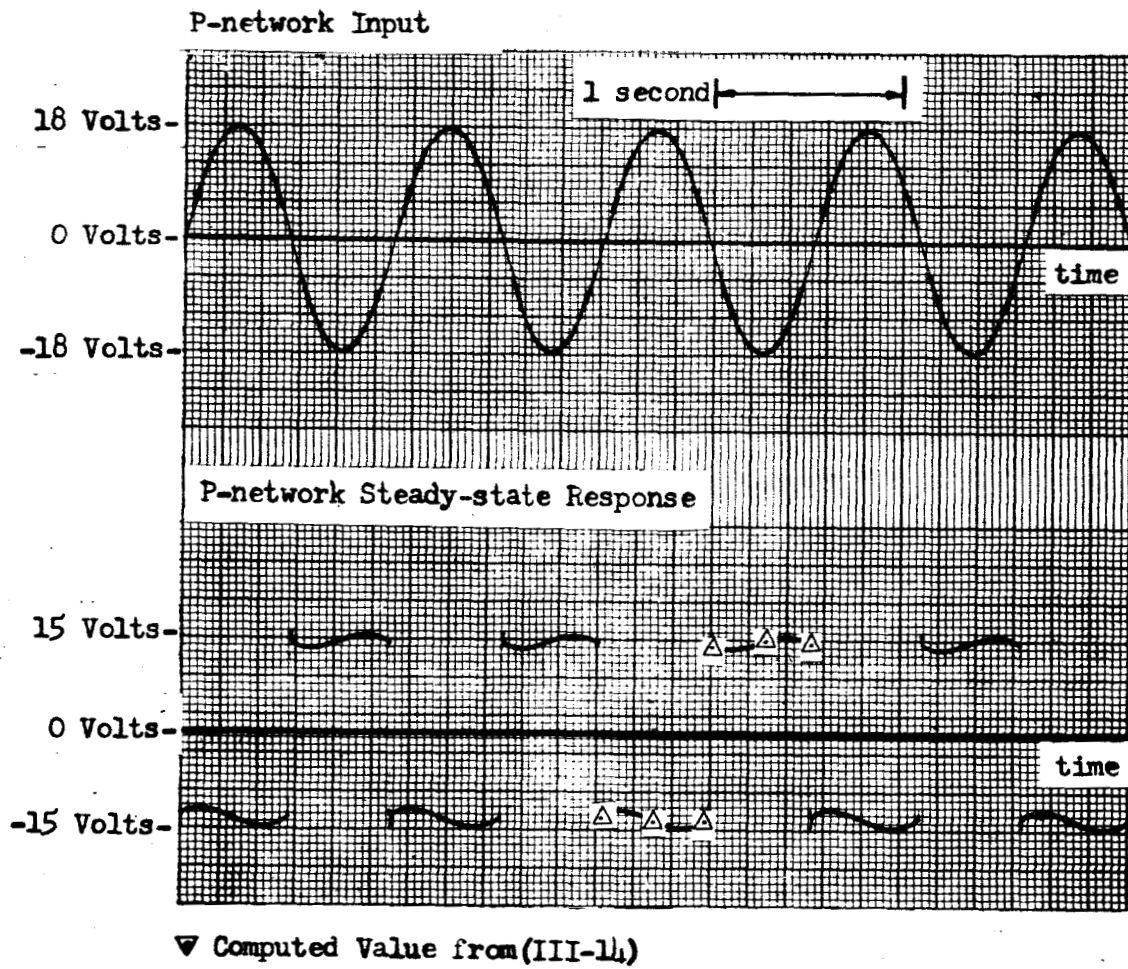


Fig. 3.-Steady-state Response of P-network

discontinuities in any period, it may be represented by a Fourier series expansion; or,

$$e_o(t) = B_o + \sum_{\ell} A_{\ell} \sin \ell \omega t + \sum_{\ell} B_{\ell} \cos \ell \omega t, \quad (\text{III-15})$$

$\ell = \text{all c. no.} \quad \ell = \text{all c. no.}$

where B_o , A_{ℓ} , and B_{ℓ} are the trigonometric Fourier coefficients.

The steady-state response has no dc component and has half-wave symmetry. Therefore, it contains no even harmonics, and only the odd harmonic coefficients need be considered. These coefficients may be determined as follows:

$$A_{\ell} = \frac{1}{T} \int_0^T e_o(t) \sin \ell \omega t \, dt \quad (\text{III-16})$$

$$B_{\ell} = \frac{1}{T} \int_0^T e_o(t) \cos \ell \omega t \, dt \quad (\text{III-17})$$

By the substitution of $e_o(t)$ from (III-14) into (III-16) and (III-17), the Fourier coefficients are evaluated as:

$$A_1 = \frac{8\beta\omega(e^{-\frac{T}{a^2}} + 1)}{T(e^{-\frac{T}{a^2}} - 1)(a^2 + \omega^2)} + \alpha \quad (\text{III-18})$$

$$A_{\ell} = \frac{8\beta\ell\omega(e^{-\frac{T}{a^2}} + 1)}{T(e^{-\frac{T}{a^2}} - 1)(a^2 + \ell^2\omega^2)} \quad \text{if } \ell \text{ is an odd integer } > 1$$

$$B_1 = \frac{8\beta a(e^{-\frac{aT}{2}} + 1)}{T(e^{-\frac{aT}{2}} - 1)(a^2 + \omega^2)} + \beta \quad (\text{III-19})$$

$$\beta_\ell = \frac{8\beta a(e^{-\frac{aT}{2}} + 1)}{T(e^{-\frac{aT}{2}} - 1)(a^2 + \ell^2 \omega^2)} \quad \text{if } \ell \text{ is an odd integer } > 1.$$

1. Optimum Value of $\frac{R_f}{R_i}$

Suppose the input $e_i(t)$ is in phase with the commutation function, or $\phi = 0^\circ$; and, it is desired that R , (the amplitude of $e_i(t)$), be equal to $-A_1$, and that B_1 , (the quadrature component of the output), be made as small as possible.

From (III-19), it can be seen that B_1 can be made as small as desired by reducing the magnitude of $a = \frac{1}{R_f C}$, or with $\frac{R_f}{R_i} = \frac{\pi^2}{8}$,

$$\lim_{a \rightarrow 0} B_1 = \frac{a R R_f}{\omega R_i} \left(1 - \frac{8}{\pi^2}\right) \bigg|_{\frac{R_f}{R_i} = \frac{\pi^2}{8}} \approx \lim_{a \rightarrow 0} \left[0.23 \frac{a R}{\omega} \right] = 0 \quad (\text{III-20})$$

As can be seen in (III-18), the magnitude of A_1 as $a \rightarrow 0$ is

$$\lim_{a \rightarrow 0} A_1 = - \frac{8 R_f R}{\pi^2 R_i} \quad (\text{III-21})$$

Then if it is desired that $A_1 = -R$, the input and feedback resistors must be adjusted such that $\frac{R_f}{R_i} = \frac{\pi^2}{8} \approx 1.23$.

It should be noted that in (III-21), the magnitude A_1 is insensitive to change in ω . However, if $a = 0$, the output of the commutated network would never reach steady-state. Thus, (III-20) and (III-21) can only be regarded as approximations whose accuracy increases as $a = \frac{1}{R_f C}$ decreases in magnitude.

Table I lists various values of A_1 and B_1 as an example to illustrate the magnitude of the errors involved in using approximations (III-20) and (III-21) for a given set of conditions.

2. Use of RC Commutated Network For Attenuating Low Frequency Inputs

Another useful property of the RC commutated network is that it can be used to attenuate input signals which are lower in frequency than the commutating function.

It can be seen in (III-10) that as ω , the input frequency, approaches zero, the steady-state dc response of the RC commutated network approaches zero, or

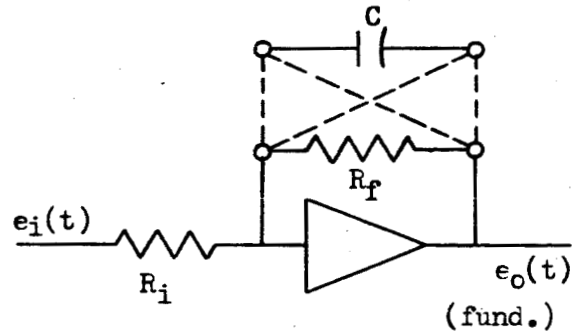
$$\lim_{\substack{a \rightarrow 0 \\ \omega \rightarrow 0}} e(t) = 0 \quad (\text{III-22})$$

$$\frac{R_f}{R_i} = 1.23$$

$$e_i(t) = 1 \sin 5.65t$$

$$e_o(t) = A_1 \sin 5.65t + B_1 \cos 5.65t$$

(fund.)



$a = \frac{1}{R_f C}$	A_1	B_1
2.9	-1.01083	0.11240
2.5	-1.00744	0.09839
2.1	-1.00447	0.08375
1.7	-1.00195	0.06856
1.3	-0.99992	0.05289
0.9	-0.99841	0.03686
0.5	-0.99744	0.02056

Table 1.-Computed Values of A_1 and B_1 Illustrating Error Magnitudes in Approximations (III-20) and (III-21) for Various Values of $a = 1/R_f C$

Therefore, (III-22) can be used as an approximation, for low frequency inputs. For example, consider the RC commutated network with

$$R_f = 5 \text{ m}$$

$$R_i = 5 \text{ m}$$

$$C = 0.1 \text{ } \mu\text{f}$$

and the frequency of commutation 0.9 hz.. The steady-state response of this configuration was investigated experimentally for a sinusoidal input with a peak-to-peak amplitude of 36 volts and frequency of 0.1 hz., and it was found that the output component at 0.1 hz. had a peak-to-peak amplitude of approximately 1.0 volt. This is an indication of the magnitude of error involved in the approximation given by (III-22).

E. Conclusions

An approximate steady-state output expression is derived for the RC commutated network with input and commutation frequencies equal. The response time of the network is directly proportional to $\frac{1}{a} = R_f C$, the product of the feedback capacitance and resistance. The steady-state output can then be expressed as a Fourier series containing only the odd harmonic terms. The amplitude of the fundamental becomes less sensitive to changes in ω as the magnitude of "a" is decreased, and the quadrature component of the fundamental can be made as small as desired by decreasing the value of "a". Also, the RC commutated network can be used for attenuating inputs with frequencies much lower than the commutation frequency.

IV. USE OF RC COMMUTATED NETWORKS FOR ELIMINATING QUADRATURE BENDING MODE SIGNAL

Several schemes are investigated for eliminating the quadrature components of the first bending mode signals in the rate gyro blender, and the effectiveness of each scheme is discussed.

A. Introduction

Several modified blender configurations utilizing RC commutated networks were tested to determine their effectiveness in removing the quadrature components of the first bending mode signals.

The forward and aft first bending mode signals were simulated by a signal generator operating at 0.9 hz., which is at the center of the frequency range of the fixed bandpass filters in the rate gyro blender. A phase shifting circuit was used to simulate the phase differences of 165° or 180° between the forward and aft first bending mode signals. The amplitudes used for the simulated first bending mode signals were 36 volts and 18 volts peak-to-peak.

In each test, the blender modification under consideration was subjected to the various phase and amplitude combinations, and the amplitude, E_0 , of the fundamental frequency in the output was recorded. A twin-T filter with center frequency at 0.9 hz. was used at the output to separate the fundamental frequency from the harmonics.

Even harmonics are generated by the absolute value circuits, and the odd harmonics are generated by the RC commutated networks.

The closed-loop time response of the system was investigated for each modification by recording the output of the integrator immediately after the blender (with a 36 volt peak-to-peak signal on both channels) was placed in the "caged" (operate) configuration. An effort was made to maintain, as nearly as possible, similar time responses for each modification. In each test, the integrator input potentiometer R_2 (used to adjust the gain) and the magnitude of $a = \frac{1}{R_f C}$ of the RC commutated networks were adjusted such that the integrator output reached 63 percent of its final value in at least one second. Because "a" is finite, there is always a finite quadrature component on the output of each commutated network.

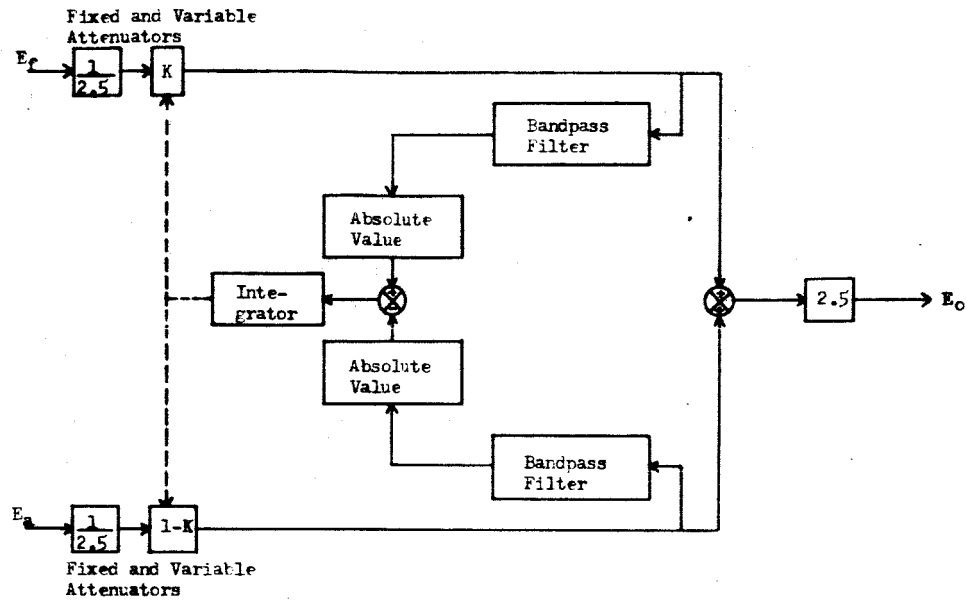
The unmodified blender configuration was tested first in order to establish a basis of comparison for each of the modifications which follow.

B. Unmodified Rate Gyro Blender

The unmodified rate gyro blender was subjected to the previously described combinations of test inputs, and R_2 of the integrator circuit was adjusted for the desired system time response. The test results appear in Figure 4.

In theory, the output of the blender, with inputs 180° out of phase, should be zero for all combinations of input amplitudes. A dc integrator bias adjustment is provided for this purpose. However, it was necessary to readjust the bias each time the input amplitudes

TEST CONFIGURATION



TEST DATA

E_r (Volts P-P)	E_a (Volts P-P)	Frequency (Hz.)	Phase Difference (Degrees)	E_o (Volts P-P)
36	36	0.9	180	0.19
18	36	0.9	180	0.20
36	18	0.9	180	0.19
18	18	0.9	180	0.15
36	36	0.9	165	4.6
18	36	0.9	165	3.0
36	18	0.9	165	3.0
18	18	0.9	165	2.2

TIME RESPONSE

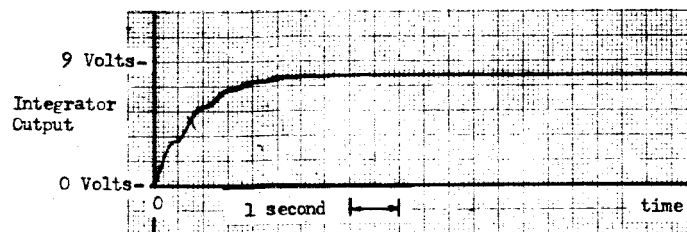


Fig. 4.-Test Data - Unmodified Rate Gyro Flender

were changed in order to maintain zero output. This problem is attributed to the nonlinear characteristics of the saturable reactors used in variable attenuators.

For the purpose of this and following tests, the bias adjustment was fixed. The value was obtained by minimizing the blender outputs for all possible combinations of forward and aft input signal amplitudes.

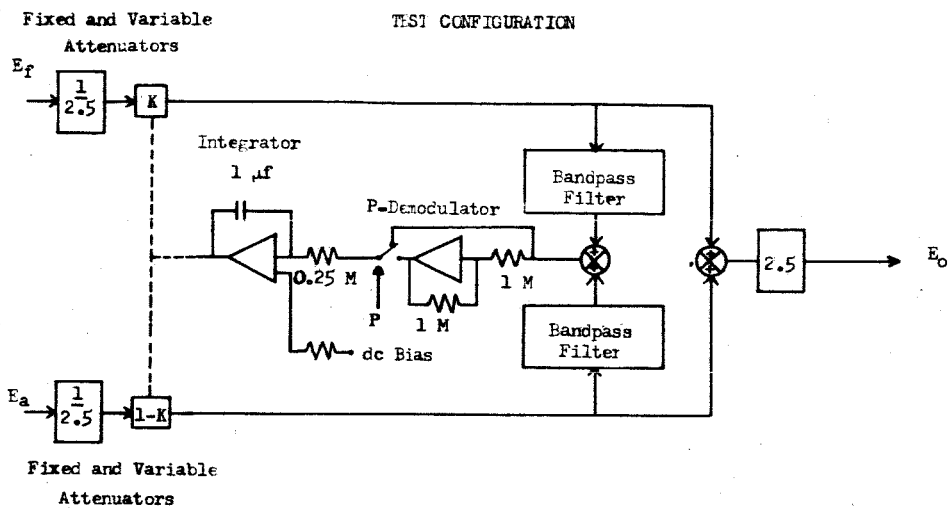
C. Modification I

The circuit of modification I is illustrated in Figure 5. This modification uses a square-wave demodulator in place of the absolute value circuits. The outputs of the bandpass filters are summed and multiplied by a square-wave function, and the resulting signal is applied to an integrator external to the blender breadboard (the breadboard integrator is not accessible).

Modification I was tested as a possible alternative to the unmodified blender configuration.

1. Theory of Operation

The P-demodulation function used to operate the SPDT switch of the P-demodulator is a square wave equal in frequency to that of the bending mode signal. The phase of the P-demodulation function is adjusted for inputs 180° out of phase such that the position of the SPDT switch changes each time the P-demodulator input passes thru zero. If the input is positive, the output is equal to the input; but, if the input is negative, the output is the negative of the input. Therefore,



TEST DATA

E_f (Volts P-P)	E_a (Volts P-P)	Frequency (Hz)	Phase Difference (Degrees)	E_o (Volts P-P)
36	36	0.9	180	0.19
18	36	0.9	180	0.16
36	18	0.9	180	0.17
18	18	0.9	180	0.15
36	36	0.9	165	4.8
18	36	0.9	165	3.2
36	18	0.9	165	3.2
18	18	0.9	165	2.3

TIME RESPONSE

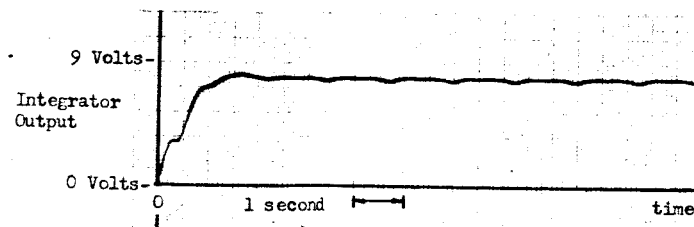


Fig. 5.-Test Data - Modification I

for inputs 180° out of phase, the P-demodulator dc output is proportional to the absolute value of the sum of the forward and aft attenuated signals.

The integrator configuration used in modification I has the same function as the integrator circuit of the unmodified blender.

2. Results

The test data and time response from the test of modification I appear in Figure 5. The results are very similar to those obtained in testing of the unmodified blender configuration.

Again, it was impossible to find one integrator bias setting which would zero the blender output for all signal combinations that were 180° out of phase.

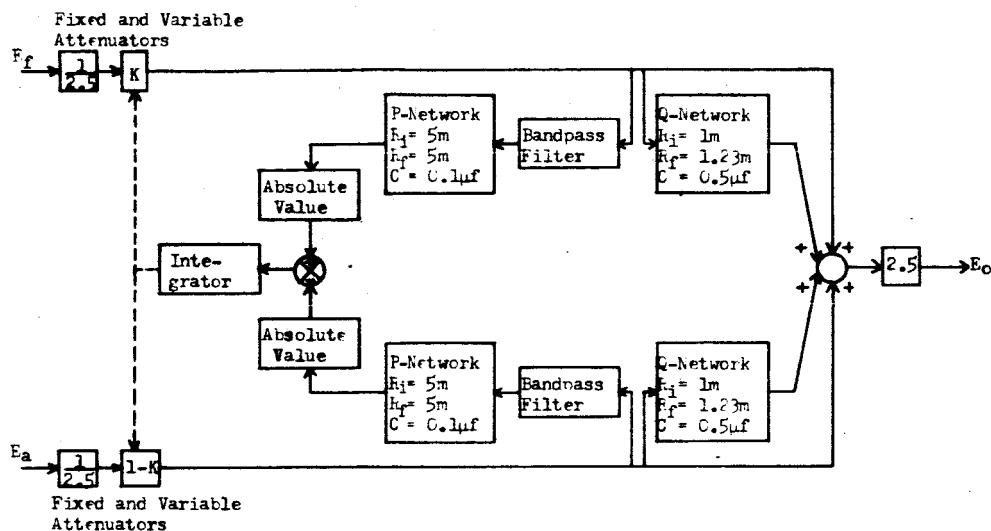
D. Modification II

The circuit configuration of modification II is illustrated in Figure 6.

The P-networks (see Figure 2) are commutated by a square-wave commutation signal at the frequency of the first bending mode. The Q-networks (see Figure 2) are commutated by a square wave commutation function in quadrature with the P commutation signal. It is not necessary for the P and Q commutation functions to have any particular phase relationship to the first bending mode signal, but their frequency must be exactly equal to the first bending mode frequency.

The P commutation function used for testing purposes was in phase with the forward first bending mode signal.

TEST CONFIGURATION



TEST DATA

E_f (Volts P-P)	E_a (Volts P-P)	Frequency (Hz)	Phase Difference (Degrees)	E_o (Volts P-P)
36	36	0.9	180	0.18
18	36	0.9	180	0.16
36	18	0.9	180	0.15
18	18	0.9	180	0.16
36	36	0.9	165	0.30
18	36	0.9	165	0.22
36	18	0.9	165	0.20
18	18	0.9	165	0.20

TIME RESPONSE

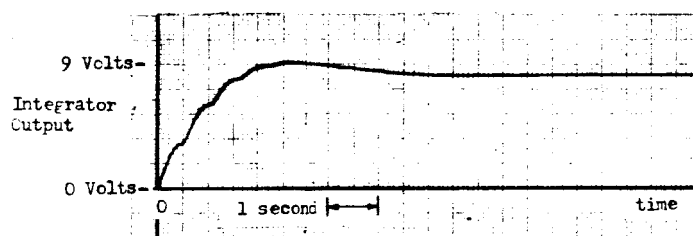


Fig. 6.-Test Data - Modification II

1. Theory of Operation

The fundamental frequency output amplitudes of the P-networks are proportional to the amplitudes of the attenuated input components that are in phase with the P commutation function; and, they are nearly independent of the quadrature input amplitudes, assuming that $a = \frac{1}{R_f C}$ is small. This can be seen from (III-20). Therefore, the integrator output increases or decreases until the amplitudes of the attenuated components of the forward and aft signals which are in phase with the P commutation function are equal.

The Q-networks are adjusted such that $\frac{R_f}{R_1} = \frac{\pi^2}{8}$. As shown in Chapter III, Section D, the fundamental output components are equal in amplitude and opposite in phase to the quadrature components of the attenuated forward and aft input signals. Then, if their outputs are summed positively with the forward and aft attenuated signals, as shown in Figure 6, the quadrature components of the blender output are cancelled.

2. Results

The test data and the time response for the circuit of modification II are given in Figure 6, and values of the commutated network components required for the given time response are given in the test configuration.

One source of error between the test results and theoretical results is the assumption concerning the magnitude of $a = \frac{1}{R_f C}$. In this configuration "a" cannot be made arbitrarily small because of the required time response of the integrator output signal.

E. Modification III

The circuit configuration of modification III is illustrated in Figure 7.

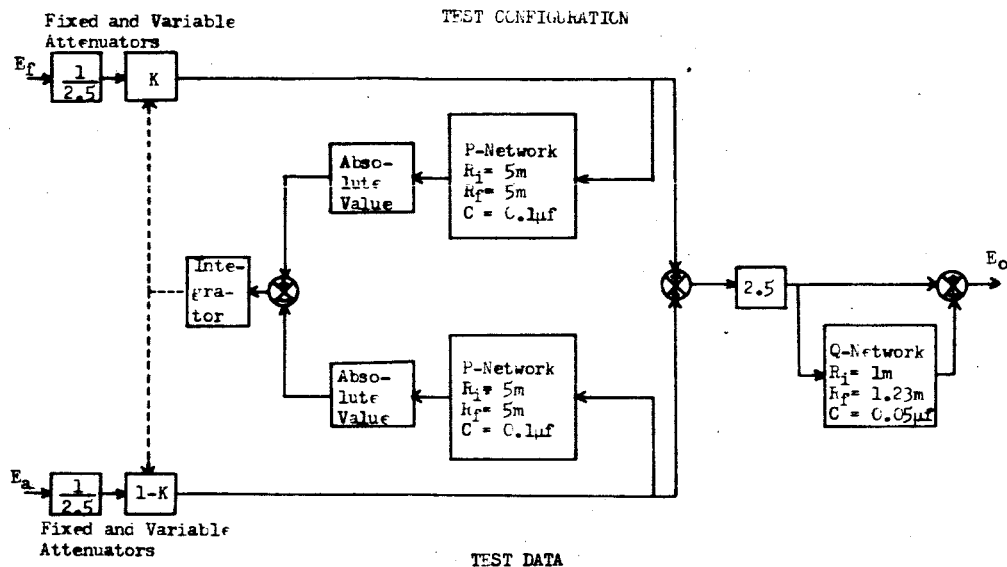
In this modification, the bandpass filters are replaced with P-networks, and a Q-network is utilized in order to remove the quadrature component of the rate gyro blender output at the first bending mode frequency.

1. Theory of Operation

The operation of modification III is similar to that of modification II.

As was shown in Chapter III, the P-network attenuates all input components whose frequencies are much less than the commutation frequency. The first bending mode frequency to be considered (which is used as the P-network commutation function), is approximately ten times the frequency of the rigid body rate signal. Then, if it is assumed that there are no components in the forward and aft input signals with frequencies higher than the first bending mode signal, the P-network can be used to perform the same function as the bandpass filter; that is, it can be used to attenuate the control signal and pass the first bending mode signal. An additional property of the P-network is that only the component of the first bending mode signal that is in phase with the P commutation function is passed.

The Q-network is adjusted such that $\frac{R_f}{R_i} = \frac{\pi^2}{8}$. Then, as shown in Chapter III, Section D, its fundamental output component is



E_f (Value P-P)	E_a (Value P-P)	Frequency (Hz)	Phase Difference (Degrees)	E_o (Volts P-P)
36	36	0.9	180	0.07
18	36	0.9	180	0.10
36	18	0.9	180	0.10
18	18	0.9	180	0.08
36	36	0.9	165	0.20
18	36	0.9	165	0.16
36	18	0.9	165	0.16
18	18	0.9	165	0.10

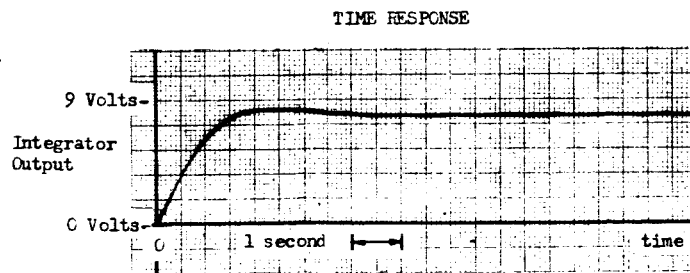


Fig. 7.-Test Data - Modification III

approximately equal in amplitude and opposite in phase to the quadrature first bending mode component of the blender output. The Q-network output, if summed with the blender output, cancels the first bending mode quadrature signal. The component values of the P and Q-networks are given in Figure 7.

2. Results

The test data and the time response of modification III appear in Figure 7.

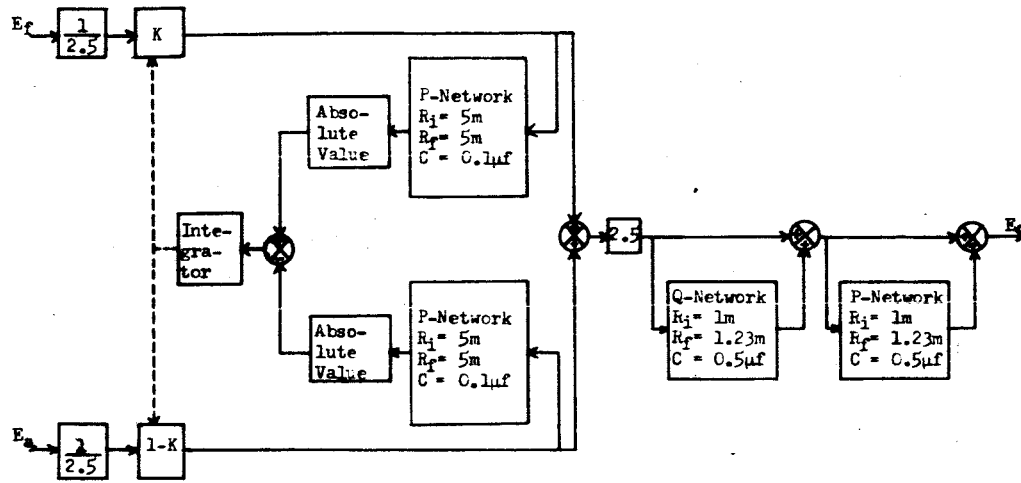
A significant improvement over the unmodified rate gyro blender configuration was realized with modification III, although a small component of the first bending mode signal in phase with the P commutation function still persisted in the output of the blender. This component is, in part, a result of the Q-network generating a small component in quadrature with the Q commutation function.

F. Modification IV

The circuit configuration of modification IV is illustrated in Figure 8.

Modifications III and IV are the same except that, in modification IV, a P-network is cascaded with the Q-network on the output of the rate gyro blender. The purposes of the additional P-network are to remove the inphase component of the first bending mode signal which is not eliminated by the rate gyro blender, and to remove the inphase component generated by the Q-network.

TEST CONFIGURATION



TEST DATA

E_r (Volts P-P)	E_a (Volts P-P)	Frequency (Hz.)	Phase Difference (Degrees)	E_o (Volts P-P)
36	36	0.9	180	0.01
18	36	0.9	180	0.01
36	18	0.9	180	0.01
18	18	0.9	180	0.008
36	36	0.9	165	0.02
18	36	0.9	165	0.015
36	18	0.9	165	0.015
18	18	0.9	165	0.01

TIME RESPONSE

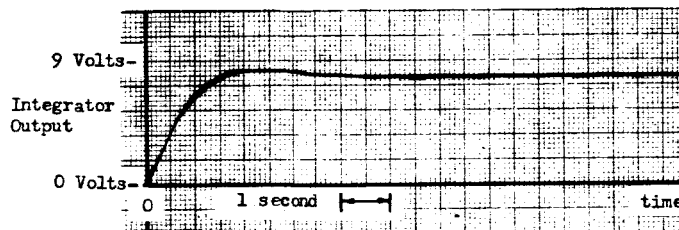


Fig. 8.-Test Data - Modification IV

1. Theory of Operation

The operation of modification IV is basically the same as that of modification III.

The P commutation function is in phase with the forward first bending mode signal, and the Q commutation function is in quadrature with the P function. The additional P-network is adjusted such that $\frac{R_f}{R_i} = \frac{\pi^2}{8}$. Then, if the output of the P-network is summed with the outputs of the blender and the Q-network, the inphase component of the blender output at the first bending mode frequency should be cancelled.

2. Results

The test data and the time response of modification IV appear in Figure 8.

As can be seen from the test data, the additional cascaded P-network greatly improves the operation of the rate gyro blender. This configuration has the added advantage of providing two methods for eliminating the inphase components of the first bending mode signals: the variable attenuators, and the cascaded P-network.

G. Conclusions

The following paragraphs briefly summarized the conclusions of Chapter IV.

Each of the modified rate gyro blender configurations investigated in this chapter require that a commutation function be available with a frequency identical to that of the first bending mode signal. In

addition, it is necessary that the P and Q commutation functions have a quadrature phase relationship. The four modifications investigated are based upon these assumptions.

The unmodified configuration is incapable of removing any quadrature components in the first bending mode signals.

The circuit of modification I yields comparable results to those of the unmodified configuration, which indicates that the P-demodulator could be used to replace the absolute value circuits.

Modification II, which utilizes P and Q-networks, eliminates the quadrature components of the first bending mode signal, but adds an additional inphase component, because the magnitude of $a = \frac{1}{R_f C}$ is not zero. The choice for the value of $a = \frac{1}{R_f C}$ involves a compromise between response time and the residual bending mode signal in the output.

In modifications III and IV, the bandpass filters are replaced by P-networks. The P-networks are used for the purpose of isolating the components of the first bending mode signals in phase with the P commutation function. In addition, the P-networks attenuate the low frequency control signal.

Modification IV provided much better first bending mode signal cancellation than any of the other modifications. As a result of the cascaded P and Q-networks on the output of the blender configuration, the undesirable inphase signal generated by the Q-network is eliminated. This also helps to reduce error caused by undesirable disturbances of the integrator output.

V. CASCADED P AND Q-NETWORK CONFIGURATION

This chapter investigates the case wherein a P and a Q-network are cascaded, as in modification IV of Chapter IV. A time expression is obtained for the steady-state fundamental output component assuming an input of the same frequency as that of the commutation signals.

A. Introduction

In the investigation of modification IV, a question arose as to what portion of the steady-state fundamental response of the P-network results from the harmonic content of the Q-network output. Therefore, an analytical study of a cascaded P and Q-network configuration was undertaken.

The notation used in the analysis of the cascaded configuration is given in Figure 9. The form of the input function $e(t)$ is restricted to $R \sin \omega t$, a sinusoid with arbitrary amplitude R and with frequency equal to that of the commutation function.

B. Derivation of Steady-State $e_2(t)$

As shown in Chapter III, Section D, an approximate steady-state expression for $e_2(t)$ can be obtained in the form of a Fourier series. The series consists of the fundamental term plus higher odd harmonics.

At this point, it is convenient to make the assumption that the first commutation of the P-network occurs at $t = 0$. Therefore, the assumed input, $R \sin \omega t$, is in phase with the P commutation function.

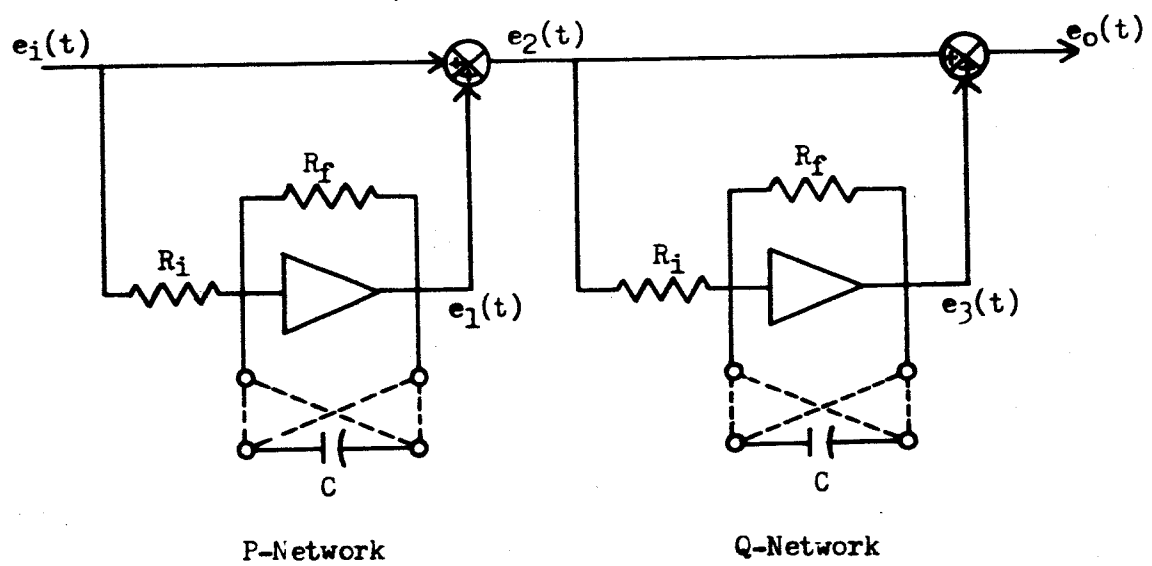


Fig. 9.-Cascaded P and Q-Network Configuration

As in the case of modification IV, the value of A_1 , the coefficient of the inphase fundamental term of the Fourier expansion of $e_1(t)$, is made equal to $-R$. This is through proper adjustment of $\frac{R_f}{R_i}$. Then, it can be seen from Figure 9, and the Fourier expansion of $e_1(t)$ in (III-15), that an approximate steady-state expression for $e_2(t)$ can be written as follows:

$$e_2(t) = \sum_{n=\text{all c. no.} > 1} A_{2n-1} \sin((2n-1)\omega t) + \sum_{n=\text{all c. no.} \geq 1} B_{2n-1} \cos((2n-1)\omega t), \quad (\text{V-1})$$

where A_{2n-1} and B_{2n-1} , the Fourier coefficients, are as given in (III-18) and (III-19).

Equation (V-1) is a steady-state approximation of $e_2(t)$, which contains an infinite number of odd harmonic terms. This equation can be used for determining an approximate steady-state response of the cascaded Q-network. First the Q-network response to a single harmonic term is determined. Then, by applying the principle of superposition to the Q-network, the total Q-network output is expressed as the sum of the responses resulting from each harmonic term in (V-1).

C. Response of Q-network to a Harmonic Input Term

Each of the harmonic terms in (V-1) can be written in generalized form as

$$e(t) = R_n \sin((2n-1)\omega t + \phi_n), \quad (\text{V-2})$$

where n is a counting number used to specify the amplitude R_n of the harmonic. The phase angle ϕ_n can be adjusted so that (V-2)

can represent the sine terms, ($\phi_n = 0^\circ$), or the cosine terms, ($\phi_n = 90^\circ$), appearing in (V-1).

Through a procedure similar to that outlined in Chapter III, the form of the output expression of the Q-network can be determined for any commutation period. The desired expression is

$$e_3(t) = Ae^{-at} + \alpha_n \sin(2n-1) \omega t + \beta_n \cos(2n-1) \omega t, \quad (V-3)$$

where

$$\alpha_n = C_{n1} \cos \phi_n - C_{n2} \sin \phi_n,$$

$$\beta_n = C_{n1} \sin \phi_n + C_{n2} \cos \phi_n,$$

$$C_{n1} = \frac{ab_n}{(2n-1)^2 \omega^2 + a^2},$$

$$C_{n2} = \frac{ab_n}{(2n-1)^2 \omega^2 + a^2},$$

$$a = \frac{1}{R_f C},$$

and

$$b_n = \frac{R_n}{R_f C}.$$

The Q-network is commutated every π/ω seconds, and the commutations of the P and Q networks are separated by $\pi/2\omega$ seconds.

It must be assumed that sufficient time after $t = 0$ has passed for the approximation of (V-1) to be valid. After the assumed elapsed

time, we apply the generalized steady-state harmonic input of (V-2) to the input of the Q-network in order to determine its output. The elapsed time is defined as kT , (where k is a counting number and T is the period of the commutation function). To simplify the analysis make the change of variable

$$t' = t - kT. \quad (V-4)$$

Then, the components of $e_2(t')$ are in phase with the corresponding terms of $e_2(t)$, and the first commutation of the Q-network occurs at $t' = \pi/2\omega$ seconds.

Assume a Q-network output, M_n volts, just prior to the first commutation. Then, the analysis proceeds as in Chapter III, Section B; that is, the value of A , the coefficient of the transient term in (V-3), is evaluated during each commutation period of the Q-network. The value of A between the first and second commutations is determined by equating $-M_n$ to the $e_3(\pi/2\omega)$ from (V-3), or

$$A = (-M_n - \alpha_n(-1)^{n-1}) e^{\frac{\pi}{2\omega}} \quad (V-5)$$

Therefore, the response of the Q-network resulting from the generalized harmonic input during the first commutation period is

$$\begin{aligned}
e_3(t') = & \left[(-M_n - (-1)^{n-1} \alpha_n) e^{-a(t' - \frac{\pi}{2\omega})} + \alpha_n \sin(2n-1) \omega t' \right. \\
& \left. + \beta_n \cos(2n-1) \omega t' \right] \cdot \\
& \left[u(t' - \frac{\pi}{2\omega}) - u(t' - \frac{3\pi}{2\omega}) \right].
\end{aligned} \tag{V-6}$$

The unit-step notation is used in (V-6) to denote the time interval over which the output expression is valid.

The foregoing procedure of determining the initial output voltage immediately following each commutation, and of equating this voltage to $e_3(t')$, evaluated from (V-3), can be continued for any period $\frac{m\pi}{\omega} + \frac{\pi}{2\omega} < t' < (m+1) \frac{\pi}{\omega} + \frac{\pi}{2\omega}$, (where m is the number of commutations minus one).

It can be shown that the output expression due to the generalized harmonic input for any period $\frac{m\pi}{\omega} + \frac{\pi}{2\omega} < t' < (m+1) \frac{\pi}{\omega} + \frac{\pi}{2\omega}$ is

$$\begin{aligned}
e_3(t') = & \left\{ (-1)^m \left[-M_n + (-1)^{n-1} \alpha_n - 2(-1)^{n-1} \alpha_n \sum_{i=0}^m e^{ai\frac{\pi}{\omega}} \right] e^{-a(t' - \frac{\pi}{2\omega})} \right. \\
& \left. + \alpha_n \sin(2n-1) \omega t' + \beta_n \cos(2n-1) \omega t' \right\} \cdot \\
& \left[u(t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega}) - u(t' - (m+1) \frac{\pi}{\omega} - \frac{\pi}{2\omega}) \right].
\end{aligned} \tag{V-7}$$

The series in (V-7) can be written in closed form by the use of (III-12). Therefore, (V-7) becomes

$$e_3(t') = \left\{ (-1)^m \left[-M_n + (-1)^{n-1} \alpha_n - 2(-1)^{n-1} \alpha_n \frac{1 - e^{a(m+1)\frac{\pi}{\omega}}}{1 - e^{\frac{a\pi}{\omega}}} \right] e^{-a(t' - \frac{\pi}{2\omega})} + \alpha_n \sin(2n-1) \omega t' + \beta_n \cos(2n-1) \omega t' \right\} \cdot \left[u(t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega}) - u(t' - (m+1)\frac{\pi}{\omega} - \frac{\pi}{2\omega}) \right] . \quad (V-8)$$

If m , the number of Q-network commutations minus one, becomes very large, the Q-network response to the generalized harmonic input term can be approximated by

$$e_3(t') = \left\{ (-1)^m (-1)^{n-1} \alpha_n \left[\frac{2e^{-at''}}{e^{-\frac{a\pi}{\omega}} - 1} \right] + \alpha_n \sin(2n-1) \omega t' + \beta_n \cos(2n-1) \omega t' \right\} \cdot \left[u(t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega}) - u(t' - (m+1)\frac{\pi}{\omega} - \frac{\pi}{2\omega}) \right] , \quad (V-9)$$

where

$$t'' = t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega} .$$

Equation (V-9) is an approximate steady-state expression for the actual Q-network response resulting from the generalized harmonic input, $R_n \sin((2n-1) \omega t' + \phi_n)$. The first term in (V-9) is an approximate square wave with frequency equal to the commutation function. The remaining terms are components at the frequency of the harmonic under consideration.

D. Steady-State Approximation of $e_3(t')$

The result of (V-9) can be extended to determine the total steady-state response of the Q-network to the input described by (V-1), since each term of (V-1) can be expressed in the form of the generalized harmonic input of (V-2).

Consider the first series, $\sum_{n=\text{all c. no.} > 1} A_{2n-1} \sin(2n-1) \omega t'$, of $e_2(t')$ obtained from (V-1). The Q-network steady-state response resulting from this series of input terms can be derived from (V-9) by letting $R_n = A_{2n-1}$ and $\phi_n = 0$. The response resulting from the terms of the second series in $e_2(t')$, $\sum_{n=\text{all c. no.}} B_{2n-1} \cos(2n-1) \omega t'$, can be derived by letting $R_n = B_{2n-1}$ and $\phi_n = 90^\circ$ in (V-9).

If the foregoing procedure is used, the resulting total steady-state approximation of $e_3(t')$ due to the steady-state input $e_2(t')$ is

$$\begin{aligned}
 e_3(t') = & \left(\sum_{n=\text{all c. no.} > 1} \left\{ (-1)^m A_{2n-1} (-1)^{n-1} \alpha_0 \left[\frac{2e^{-at''}}{e^{\frac{-a\pi}{\omega}} - 1} \right] + A_{2n-1} \left[\alpha_0 \sin(2n-1) \omega t' \right. \right. \right. \\
 & \left. \left. + \beta_0 \cos(2n-1) \omega t' \right] \right\} \\
 & + \sum_{n=\text{all c. no.}} \left\{ (-1)^m B_{2n-1} (-1)^{n-1} \alpha_{90} \left[\frac{2e^{-at''}}{e^{\frac{-a\pi}{\omega}} - 1} \right] \right. \\
 & \left. \left. + B_{2n-1} \left[\alpha_{90} \sin(2n-1) \omega t' + \beta_{90} \cos(2n-1) \omega t' \right] \right\} \right). \\
 & \left[u\left(t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega}\right) - u\left(t' - (m+1)\frac{\pi}{\omega} - \frac{\pi}{2\omega}\right) \right], \quad (V-10)
 \end{aligned}$$

where

$$t'' = t' - \frac{m\pi}{\omega} - \frac{\pi}{2\omega},$$

$$\alpha_0 = \frac{\alpha_n}{R_n} \mid \phi_n = 0^\circ,$$

$$\alpha_{90} = \frac{\alpha_n}{R_n} \mid \phi_n = 90^\circ,$$

$$\beta_0 = \frac{\beta_n}{R_n} \mid \phi_n = 0^\circ,$$

and

$$\beta_{90} = \frac{\beta_n}{R_n} \mid \phi_n = 90^\circ.$$

Equation (V-10) is an approximate steady-state expression for $e_3(t')$, the response of the Q-network resulting from the input described by (V-1). It contains an infinite number of odd harmonic terms plus a fundamental frequency component. This component consists of the sum of the contributions produced by each of the terms of (V-1).

E. Evaluation of the Coefficients of the Fundamental Terms in the Fourier Series Expression for $e_3(t')$

The approximate steady-state expression for $e_3(t')$ given by (V-10) can be expressed as a Fourier series in trigonometric form. The coefficients of the fundamental terms are

$$A'_1 = \frac{2}{T} \int_0^T e_3(t') \sin \omega t' dt',$$

and

(V-11)

$$B'_1 = \frac{2}{T} \int e_3(t') \cos \omega t' dt',$$

where the primes on A_1 and B_1 are used to avoid confusion with A_1 and B_1 in the Fourier series of $e_1(t)$.

The integration in (V-11) results in

$$\begin{aligned} A'_1 = & \sum_{\substack{n=\text{all c.} \\ \text{no.} > 1}} \frac{8 A_{2n-1} (-1)^{n-1} \alpha_0 a(e^{\frac{-a\pi}{\omega}} + 1)}{T(e^{\frac{-a\pi}{\omega}} - 1)(a^2 + \omega^2)} \\ & + \sum_{n=\text{all c. no.}} \frac{8 B_{2n-1} (-1)^{n-1} \alpha_{90} a(e^{\frac{-a\pi}{\omega}} + 1)}{T(e^{\frac{-a\pi}{\omega}} - 1)(a^2 + \omega^2)} \\ & + B_1 \alpha_{90}, \end{aligned} \quad (V-12)$$

and

$$\begin{aligned} B'_1 = & \sum_{\substack{n=\text{all c.} \\ \text{no.} > 1}} \frac{8 A_{2n-1} (-1)^{n-1} \alpha_0 \omega(e^{\frac{-a\pi}{\omega}} + 1)}{T(e^{\frac{-a\pi}{\omega}} - 1)(a^2 + \omega^2)} \\ & + \sum_{\substack{n=\text{all c.} \\ \text{no.}}} \frac{8 B_{2n-1} (-1)^{n-1} \alpha_{90} \omega(e^{\frac{-a\pi}{\omega}} + 1)}{T(e^{\frac{-a\pi}{\omega}} - 1)(a^2 + \omega^2)} \\ & + B_1 \beta_{90} \end{aligned} \quad (V-13)$$

As can be seen in (V-12) and (V-13), the individual terms of A'_1 and B'_1 resemble B_1 and A_1 in form.

The coefficients of the harmonic terms of $e_3(t')$ can be derived in a similar manner if necessary. However, we are interested only in the fundamental coefficients in this case.

F. Evaluation of the Coefficients of the Fundamental Terms In
The Fourier Series Expression for $e_o(t')$

As in the case of modification IV, the value of $\frac{R_f}{R_i}$ is adjusted to approximately 1.23. Then the component of B'_1 resulting from the B_1 input term at the Q-network is cancelled in the output summing junction, and the fundamental coefficients of $e_o(t')$, which will be denoted by A''_1 and B''_1 , are

$$A''_1 = A'_1$$

and

(V-14)

$$B''_1 = B'_1 + B_1 .$$

Furthermore, the fundamental frequency term of $e_o(t')$ can be expressed as follows:

$$e_o(t') = A''_1 \sin \omega t' + B''_1 \cos \omega t' . \quad (V-15)$$

(fund.)

As can be seen in (V-12), (V-13), and (V-14), the expressions for A''_1 and B''_1 contain several infinite series. Therefore, the exact computation of A''_1 and B''_1 is not possible unless the series can be written in closed form. However, these expressions answer the question of what portion of A''_1 and B''_1 is contributed by each of the harmonic terms of $e_2(t')$.

The expressions for A''_1 and B''_1 are independent of M_n , the initial Q-network output voltage just prior to the first commutation. Therefore, the steady-state expression for $e_o(t')$ is not dependent upon the time at which $e_2(t')$ is applied to the Q-network.

G. Conclusions

The steady-state response of the P-network contains a fundamental frequency term plus an infinite number of odd harmonic terms. Each of these harmonic terms, when applied to the Q-network, generates a Q-network steady-state response term at the fundamental frequency. If the assumption is made that the principle of superposition applies to the Q-network, the Q-network response at the fundamental frequency is the sum of the contributions of the individual input components. The steady-state Q-network response is independent of the initial output voltage at the first commutation instant, and the derived Q-network steady-state response can be used to determine the fundamental frequency component of the cascaded configuration output.

IV. RECOMMENDATIONS

The derived P-network steady-state output expression should be investigated further to determine a Fourier series representation in the case wherein the input and commutation frequencies are different. The frequency response of the P-network might then be determined analytically.

Also, the use of multiple capacitor RC commutated networks should be investigated in the various blender configurations. The amplitudes of the undesirable output harmonics of the RC commutated networks could be reduced in this way.

The cascaded P and Q-network configuration with multiple commutated capacitors might be investigated as an alternate to the rate gyro blender.

REFERENCES

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